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Updated estimate of the muon magnetic moment using revised results from e^+e^- annihilation

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Abstract. A new evaluation of the hadronic vacuum polarization contribution to the muon magnetic moment is presented. We take into account the reanalysis of the low-energy e^+e^- annihilation cross section into hadrons by the CMD-2 Collaboration. The agreement between e^+e^- and τ spectral functions in the $\pi\pi$ channel is found to be much improved. Nevertheless, significant discrepancies remain in the center-of-mass energy range between 0.85 and 1.0 GeV, so that we refrain from averaging the two data sets. The values found for the lowest-order hadronic vacuum polarization contributions are

$$a_\mu^{\rm had,LO} = \begin{cases} (696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad})\,10^{-10} & [e^+e^- -\text{based}], \\ (711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{\rm SU(2)})\,10^{-10} & [\tau-\text{based}], \end{cases}$$

where the errors have been separated according to their sources: experimental, missing radiative corrections in e^+e^- data, and isospin breaking. The corresponding Standard Model predictions for the muon magnetic anomaly read

$$a_\mu = \begin{cases} (11\,659\,180.9 \pm 7.2_{\rm had} \pm 3.5_{\rm LBL} \pm 0.4_{\rm QED+EW})\,10^{-10} & [e^+e^- -\text{based}], \\ (11\,659\,195.6 \pm 5.8_{\rm had} \pm 3.5_{\rm LBL} \pm 0.4_{\rm QED+EW})\,10^{-10} & [\tau-\text{based}], \end{cases}$$

where the errors account for the hadronic, light-by-light (LBL) scattering and electroweak contributions. The deviations from the measurement at BNL are found to be $(22.1 \pm 7.2 \pm 3.5 \pm 8.0)\,10^{-10}$ $(1.9\ \sigma)$ and $(7.4 \pm 5.8 \pm 3.5 \pm 8.0)\,10^{-10}$ $(0.7\ \sigma)$ for the e^+e^-- and τ-based estimates, respectively, where the second error is from the LBL contribution and the third one from the BNL measurement.

1 Introduction

Hadronic vacuum polarization in the photon propagator plays an important role in the precision tests of the Standard Model. This is the case for the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$ where the hadronic vacuum polarization component, computed from experimentally determined spectral functions, is the leading contributor to the uncertainty of the theoretical prediction.

Spectral functions are obtained from the cross sections for e^+e^- annihilation into hadrons. The accuracy of the calculations has therefore followed the progress in the quality of the corresponding data [1]. Because the latter was not always suitable, it was deemed necessary to resort to other sources of information. One such possibility was the use of the vector spectral functions [2] derived from the study of hadronic τ decays [3] for the energy range less than $m_\tau c^2 \sim 1.8$ GeV. Also, it was demonstrated that perturbative QCD could be applied to energy scales as low as 1-2 GeV [4], thus offering a way to replace poor e^+e^- data in some energy regions by a reliable and precise theoretical prescription [5–9].

A complete analysis including all available experimental data was presented in [10], taking advantage of the new precise results in the $\pi\pi$ channel from the CMD-2 experiment [11] and from the ALEPH analysis of τ decays [12], and benefiting from a more complete treatment of isospin-breaking corrections [13, 14]. In addition to these major updates, the contributions of the many exclusive channels

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up to 2 GeV center-of-mass energy were completely revisited. It was found that the e^+e^- and the isospin-breaking corrected τ spectral functions were not consistent within their respective uncertainties, thus leading to inconsistent predictions for the lowest-order hadronic contribution to the muon magnetic anomaly:

$$a_\mu^{\rm had,LO} = \begin{cases} (684.7 \pm 6.0_{\rm exp} \pm 3.6_{\rm rad})\, 10^{-10} \quad [e^+e^-\text{−based}], \\ (709.0 \pm 5.1_{\rm exp} \pm 1.2_{\rm rad} \pm 2.8_{\rm SU(2)})\, 10^{-10} \quad [\tau\text{−based}], \end{cases} \tag{1}$$

The quoted uncertainties are experimental, missing radiative corrections to some e^+e^- data, and isospin breaking. The leading contribution to the discrepancy originated in the $\pi\pi$ channel with a difference of $(-21.2 \pm 6.4_{\rm exp} \pm 2.4_{\rm rad} \pm 2.6_{\rm SU(2)}\,(\pm 7.3_{\rm total}))\,10^{-10}$. The estimate based on e^+e^- data has been confirmed by another analysis using the same input data [15]. When compared to the world average of the muon magnetic anomaly measurements,

$$a_\mu = (11\,659\,203 \pm 8)\, 10^{-10}, \tag{2}$$

which is dominated by the 2002 BNL result using positive muons [16], the respective e^+e^--based and τ-based predictions disagreed at the 3.0 and 0.9 σ level, respectively, when adding experimental and theoretical errors in quadrature.

The purpose of this letter is to update our analysis [10] in light of the following developments.

- The CMD-2 Collaboration at Novosibirsk discovered that part of the radiative treatment was incorrectly applied to the data. A complete reanalysis has been carried out and presented for publication [17]. As the CMD-2 data dominate the e^+e^--based prediction (1), the changes produce a significant effect in the final result. Recently available results from the SND Collaboration are also included.
- No significant change occurred for the τ-based prediction. The only relevant fact is a new result [18] for the branching ratio of the $\tau^- \to \nu_\tau h^-\pi^0$ mode (h^- stands for a charged pion or kaon).

2 Muon magnetic anomaly

It is convenient to separate the Standard Model (SM) prediction for the anomalous magnetic moment of the muon into its different contributions,

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm weak}, \tag{3}$$

with

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}, \tag{4}$$

and where $a_\mu^{\rm QED} = (11\,658\,470.6 \pm 0.3)\,10^{-10}$ is the pure electromagnetic contribution (see [19, 20] and references therein[1]), $a_\mu^{\rm had,LO}$ is the lowest-order contribution from hadronic vacuum polarization, $a_\mu^{\rm had,HO}$=$(-10.0\pm0.6)10^{-10}$ is the corresponding higher-order part [2, 23], and $a_\mu^{\rm weak} = (15.4\pm0.1\pm0.2)10^{-10}$, where the first error is the hadronic uncertainty and the second is due to the Higgs mass range, accounts for corrections due to exchange of the weakly interacting bosons up to two loops [24]. For the light-by-light (LBL) scattering part we add the values for the pion-pole contribution [25–27] and the other terms [26, 27] to obtain $a_\mu^{\rm had,LBL} = (8.6 \pm 3.5)\,10^{-10}$.

Owing to the analyticity of the vacuum polarization correlator, the contribution of the hadronic vacuum polarization to a_μ can be calculated via the dispersion integral [28]

$$a_\mu^{\rm had,LO} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R(s), \tag{5}$$

where $K(s)$ is the QED kernel [29],

$$K(s) = x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{(1+x)}{(1-x)}x^2\ln x, \tag{6}$$

with $x = (1-\beta_\mu)/(1+\beta_\mu)$ and $\beta_\mu = (1-4m_\mu^2/s)^{1/2}$. In Eq. (5), $R(s) \equiv R^{(0)}(s)$ denotes the ratio of the 'bare' cross section for e^+e^- annihilation into hadrons to the pointlike muon-pair cross section. The 'bare' cross section is defined as the measured cross section, corrected for initial-state radiation, electron-vertex loop contributions and vacuum polarization effects in the photon propagator (note that photon radiation in the final state (FSR) is included in the 'bare' cross section). The reason for using the 'bare' (*i.e.* lowest order) cross section is that a full treatment of higher orders is anyhow needed at the level of a_μ, so that the use of 'dressed' cross sections would entail the risk of double-counting some of the higher-order contributions.

The function $K(s)$ decreases monotonically with increasing s. It gives a strong weight to the low energy part of the integral (5). About 91% of the total contribution to $a_\mu^{\rm had,LO}$ is accumulated at center-of-mass energies $\sqrt{s}$ below 1.8 GeV and 73% of $a_\mu^{\rm had,LO}$ is covered by the two-pion final state which is dominated by the $\rho(770)$ resonance.

[1] Some adjustment was recently made concerning the fourth-order contribution from the leptonic light-by-light scattering, mostly affecting the QED prediction for a_e and through it the value of α [21, 22]. The resulting change in $a_\mu^{\rm QED}$ is within the quoted uncertainty of $0.3\,10^{-10}$ and has not been included in the present analysis

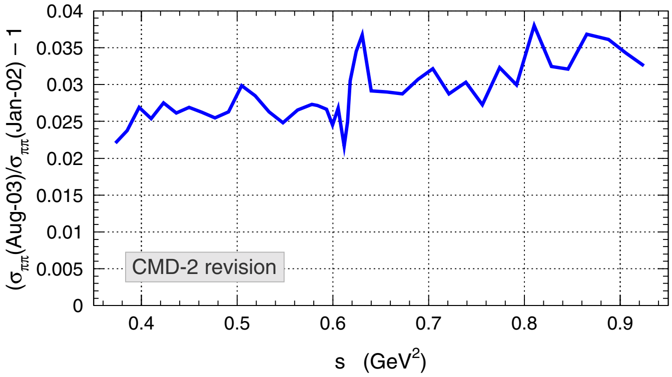

Fig. 1. Relative change in the $e^+e^- \to \pi^+\pi^-$ cross section of the revised CMD-2 analysis [17] with respect to the one previously published [11]

3 Changes to the input data

3.1 e^+e^- annihilation data

The CMD-2 data, published in 2002 for the $\pi\pi$ channel [11], have been completely reanalyzed [17] following the discovery of an incorrect implementation of radiative corrections in the analysis program. Overall, the pion-pair cross section increased by 2.1% to 3.8% in the measured energy range (*cf.* Fig. 1), well above the previously quoted total systematic uncertainty of 0.6%. Specifically, the leptonic vacuum polarization contribution in the t-channel had been inadvertently left out in the calculation of the Bhabha cross section. This effect produced a bias in the luminosity determination, varying from 2.2% to 2.7% in the 0.60-0.95 GeV energy range. The problem consequently affected the measured cross sections for all hadronic channels. Another problem was found in the radiative corrections for the muon-pair process, ranging from 1.2% to 1.4% in the same region. A more refined treatment of hadronic vacuum polarization was performed, with changes not exceeding 0.2% for most data points. The effects in the Bhabha- and muon-pair channels also affected the event separation. In the CMD-2 analysis, Bhabha events are well identified using the electron calorimeter signature while pions are not separated from muons. The numbers of electron-, muon- and pion-pair events are based on a likelihood method keeping the ratio of muons to electrons fixed through the corresponding QED cross sections. Thus, the corrections to these cross sections had an effect on the event separation and the measured ratio of pion pairs to electron and muon pairs changed by typically 0.7%.

The correction of the bias in the luminosity determination increases all hadronic cross sections published by CMD-2. The changes are 2.4% and 2.7% on the ω and ϕ resonance cross sections, respectively. They are not yet available for the energy range above the ϕ. Instead we use an estimated correction of 1.7%, which insignificantly affects the contribution of the multihadron processes between 1.05 and 1.40 GeV. All these luminosity corrections have been applied to the present analysis. Also, the CMD-2 Collaboration now provides hadronic vacuum polarization-corrected ω and ϕ cross sections so that we do not apply this correction anymore.

Newly published data by SND on the ω resonance [30] and the $2\pi^+2\pi^-$ as well as $\pi^+\pi^-2\pi^0$ modes [31] (unchanged cross sections for the latter two, but reduced systematics with respect to previous publications) have been included in this update.

We refer to our previous analysis [10] for a detailed discussion of radiative corrections, in particular the effect of final-state radiation by the charged hadrons. Also given therein is a compilation of all input data used to evaluate the integral (5).

3.2 Data from hadronic τ decays

The only update here relates to the normalization of the spectral function in the $\pi\pi$ channel. New results have been presented by the L3 Collaboration on branching ratios for hadronic τ decays [18]. Their value for the $\tau^- \to \nu_\tau h^-\pi^0$ mode, $(25.89 \pm 0.16 \pm 0.10)$%, gives, after correcting for the $K^-\pi^0$ contribution [32, 33], a result of $(25.44 \pm 0.16 \pm 0.10)$% for the $\pi^-\pi^0$ mode, in agreement with the previous measurements [12, 34, 35], yielding the world average (25.46 ± 0.10)%, which is used in the present analysis.

To use the τ spectral functions in the vacuum polarization dispersion integral, a value for the CKM matrix element $|V_{ud}|$ is necessary. In the previous analysis, we used the average of two determinations [36], $|V_{ud}| = 0.9734 \pm 0.0008$ from β decays and $|V_{ud}| = 0.9756 \pm 0.0006$ from $K_{\ell 3}$ decays and CKM unitarity, which are not consistent. The final error was scaled up correspondingly. The determination of V_{us} from hyperon decays [37] is in fact more consistent with β decays, yielding from unitarity $|V_{ud}| = 0.9744 \pm 0.0006$. New information is expected from recent $K_{\ell 3}$ and neutron decay experiments. For the moment we keep our previous average, $|V_{ud}| = 0.9748 \pm 0.0010$, since the enlarged error covers the range of measured values. The V_{ud} uncertainty corresponds to a shift of the τ-based a_μ^{had} estimate of $1.1\ 10^{-10}$, which is small compared to the total uncertainty of $5.8\ 10^{-10}$.

4 Comparison of e^+e^- and τ spectral functions

The new e^+e^- and the isospin-breaking corrected τ spectral functions can be directly compared for the $\pi\pi$ final state. The τ spectral function is obtained by averaging ALEPH [3], CLEO [38] and OPAL [39] results [10]. The e^+e^- data are plotted as a point-by-point ratio to the τ spectral function in Fig. 2, in a wide energy range (upper plot) and in the region around the ρ peak (lower plot). The central band in Fig. 2 give the quadratic sum of the statistical and systematic errors of the τ spectral function obtained by combining all τ data. The e^+e^- data have moved closer to the τ results: they are now consistent below and around the peak, while, albeit reduced, the discrepancy persists for energies larger than 0.85 GeV.

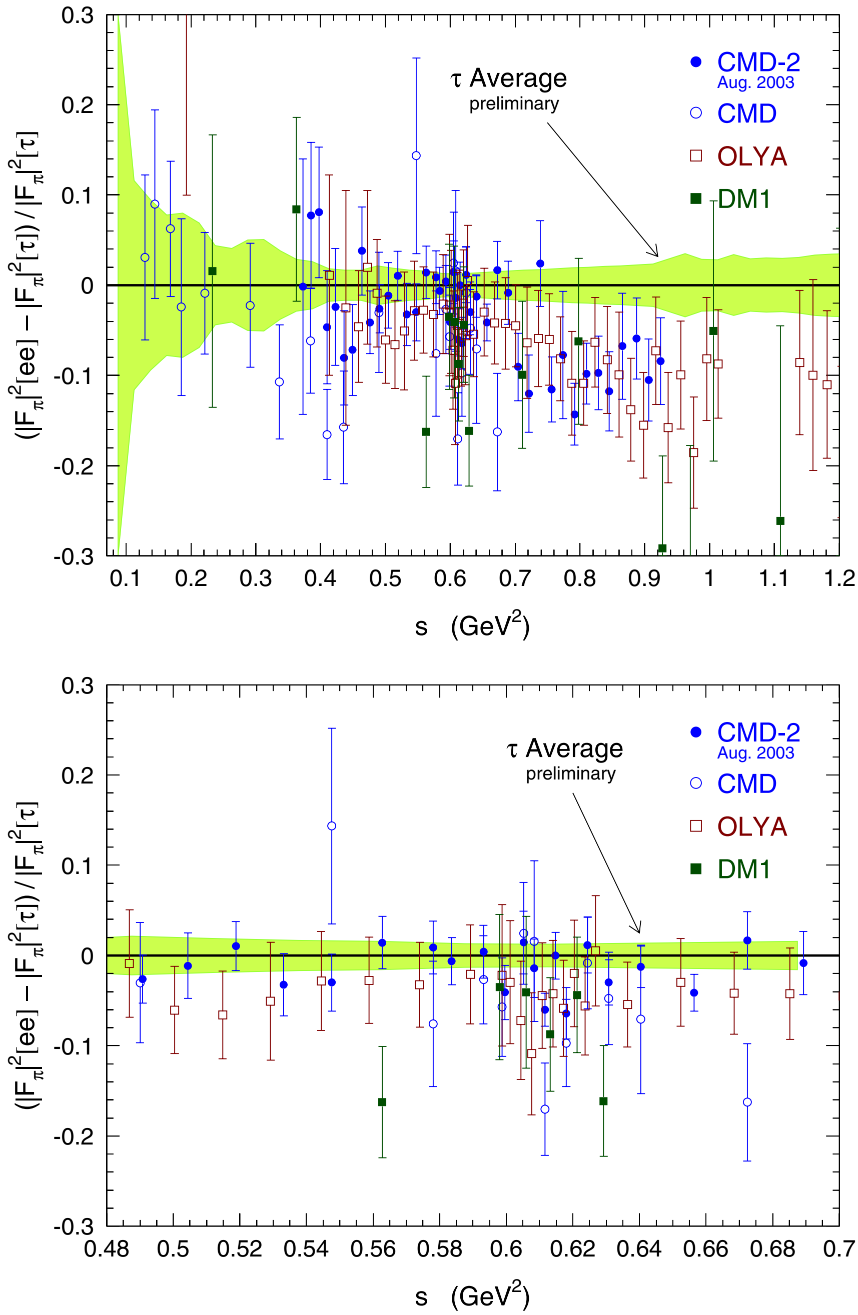

Fig. 2. Relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- and isospin-breaking corrected τ data, expressed as a ratio to the τ spectral function. The band shows the uncertainty on the latter. The e^+e^- data are from CMD-2 [17], CMD [40], OLYA [40,41] and DM1 [42]. The lower figure emphasizes the ρ peak region

A convenient way to assess the compatibility between e^+e^- and τ spectral functions proceeds with the evaluation of τ decay fractions using the relevant e^+e^- spectral functions as input. All the isospin-breaking corrections detailed in [10] are included. This procedure provides a quantitative comparison using a single number. The weighting of the spectral function is however different from the vacuum polarization kernels. Using the branching fraction $\mathcal{B}(\tau^- \to \nu_\tau e^- \bar{\nu}_e) = (17.810 \pm 0.039)\%$, obtained assuming leptonic universality in the charged weak current [12], the prediction for the $\pi\pi$ channel is

$$\mathcal{B}_{\rm CVC}(\tau^- \to \nu_\tau \pi^- \pi^0) = (24.52 \pm 0.26_{\rm exp} \pm 0.11_{\rm rad} \pm 0.12_{\rm SU(2)})\%, \quad (7)$$

where the errors quoted are split into uncertainties from the experimental input (the e^+e^- annihilation cross sections) and the numerical integration procedure, the missing radiative corrections applied to the relevant e^+e^- data, and the isospin-breaking corrections when relating τ and e^+e^- spectral functions. Even though the corrections to the CMD-2 results have reduced the discrepancy be-

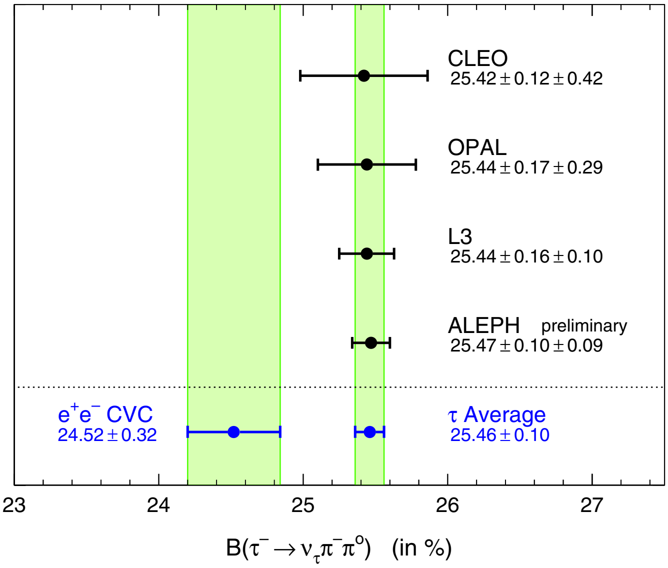

Fig. 3. The measured branching ratios for $\tau^- \to \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \to \pi^+\pi^-$ spectral function applying the isospin-breaking correction factors discussed in [10]. The measured branching ratios are from ALEPH [12], CLEO [34] and OPAL [35]. The L3 and OPAL results are obtained from their $h\pi^0$ branching ratio, reduced by the small $K\pi^0$ contribution measured by ALEPH [32] and CLEO [33]

tween (7) and the world average of the direct $\mathcal{B}(\tau^- \to \nu_\tau \pi^- \pi^0)$ measurements (*cf.* Section 3.2) from 4.6 to 2.9 standard deviations (adding all errors in quadrature), the remaining difference of $(-0.94 \pm 0.10_\tau \pm 0.26_{\rm ee} \pm 0.11_{\rm rad} \pm 0.12_{\rm SU(2)} (\pm 0.32_{\rm total}))\%$ is still problematic. Since the disagreement between e^+e^- and τ spectral functions is more pronounced at energies above 850 MeV, we expect a smaller discrepancy in the calculation of $a_\mu^{\rm had,LO}$ because of the steeply falling function $K(s)$. More information on the comparison is displayed in Fig. 3 where it is clear that ALEPH, CLEO, L3 and OPAL all separately, but with different significance, disagree with the e^+e^--based CVC result.

5 Results

The integration procedure and the specific contributions – near $\pi\pi$ threshold, the ω and ϕ resonances, the narrow quarkonia and the high energy QCD prediction – are treated as in our previous analysis [10]. The contributions from the different processes in their indicated energy ranges are listed in Table 1. Wherever relevant, the two e^+e^-- and τ-based evaluations are given. The discrepancies among them discussed above are now expressed in terms of $a_\mu^{\rm had,LO}$, giving smaller estimates for the e^+e^--based data set by $(-11.9 \pm 6.4_{\rm exp} \pm 2.4_{\rm rad} \pm 2.6_{\rm SU(2)} (\pm 7.3_{\rm total}))\,10^{-10}$ for the $\pi\pi$ channel and $(-2.8 \pm 2.6_{\rm exp} \pm 0.3_{\rm rad} \pm 1.0_{\rm SU(2)} (\pm 2.9_{\rm total}))\,10^{-10}$ for the sum of the 4π channels. The total discrepancy $(-14.7 \pm 6.9_{\rm exp} \pm 2.7_{\rm rad} \pm 2.8_{\rm SU(2)} (\pm 7.9_{\rm total}))\,10^{-10}$ amounts to 1.9 standard deviations. The difference within errors could now be considered to be acceptable, however the systematic disagreement between the e^+e^- and $\tau\,\pi\pi$ spectral functions at high energies precludes one from performing a straightforward combination of the two evaluations.

5.1 Results for a_μ

The results for the lowest order hadronic contribution are

$$a_\mu^{\rm had,LO} = (696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad})\,10^{-10} \quad [e^+e^- - \text{based}],$$
$$a_\mu^{\rm had,LO} = (711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{\rm SU(2)})\,10^{-10} \quad [\tau - \text{based}]. \tag{8}$$

Adding to these the QED, higher-order hadronic, light-by-light scattering and weak contributions as given in Section 2, we obtain the SM predictions

$$a_\mu^{\rm SM} = (11\,659\,180.9 \pm 7.2_{\rm had} \pm 3.5_{\rm LBL} \pm 0.4_{\rm QED+EW})\,10^{-10} \quad [e^+e^- - \text{based}],$$
$$a_\mu^{\rm SM} = (11\,659\,195.6 \pm 5.8_{\rm had} \pm 3.5_{\rm LBL} \pm 0.4_{\rm QED+EW})\,10^{-10} \quad [\tau - \text{based}]. \tag{9}$$

These values can be compared to the present measurement (2). Adding experimental and theoretical errors in quadrature, the differences between measured and computed values are found to be

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (22.1 \pm 7.2_{\rm had,LO} \pm 3.5_{\rm other} \pm 8.0_{\rm exp})\,10^{-10} \quad [e^+e^- - \text{based}],$$
$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (\ \ 7.4 \pm 5.8_{\rm had,LO} \pm 3.5_{\rm other} \pm 8.0_{\rm exp})\,10^{-10} \quad [\tau - \text{based}], \tag{10}$$

where the first error quoted is specific to each approach, the second is due to contributions other than hadronic vacuum polarization, and the third is the BNL g-2 experimental error. The last two errors are identical in both evaluations. The differences (10) correspond to 1.9 and 0.7 standard deviations, respectively. A graphical comparison of the results (9) with the experimental value is given in Fig. 4. Also shown are our estimates [1,9], obtained before the CMD-2 and the new τ data were available (see discussion below), and the e^+e^--based evaluations of [10,15], obtained with the previously published, uncorrected CMD-2 data [11].

6 Discussion

Although the new corrected CMD-2 $\pi^+\pi^-$ results are now consistent with τ data for the energy region below 850 MeV, the remaining discrepancy for larger energies is unexplained at present. Hence, one could question the validity of either e^+e^- data with their large radiative corrections, τ data, or the isospin-breaking corrections applied to τ data. We shall briefly discuss these points below.

Table 1. Summary of the $a_\mu^{\rm had,LO}$ contributions from e^+e^- annihilation and τ decays. The uncertainties on the vacuum polarization and FSR corrections are given as second errors in the individual e^+e^- contributions, while those from isospin breaking are similarly given for the τ contributions. These 'theoretical' uncertainties are correlated among all channels, except in the case of isospin breaking which shows little correlation between the 2π and 4π channels. The errors given for the sums in the last line are from the experiment, the missing radiative corrections in e^+e^- and, in addition for τ, SU(2) breaking

Modes	Energy [GeV]	$a_\mu^{\rm had,LO}$ (10^{-10}) e^+e^-	τ [1]	$\Delta(e^+e^- - \tau)$
Low s exp. $\pi^+\pi^-$	$[2m_{\pi^\pm}$–0.500]	$58.04 \pm 1.70 \pm 1.17$	$56.03 \pm 1.60 \pm 0.28$	$+2.0 \pm 2.6$
$\pi^+\pi^-$	[0.500–1.800]	$450.16 \pm 4.89 \pm 1.57$	$464.03 \pm 2.95 \pm 2.34$	-13.9 ± 6.4
$\pi^0\gamma$, $\eta\gamma$ [2]	[0.500–1.800]	$0.93 \pm 0.15 \pm 0.01$	–	–
ω	[0.300–0.810]	$37.96 \pm 1.02 \pm 0.31$	–	–
$\pi^+\pi^-\pi^0$ [below ϕ]	[0.810–1.000]	$4.20 \pm 0.40 \pm 0.05$	–	–
ϕ	[1.000–1.055]	$35.71 \pm 0.84 \pm 0.20$	–	–
$\pi^+\pi^-\pi^0$ [above ϕ]	[1.055–1.800]	$2.45 \pm 0.26 \pm 0.03$	–	–
$\pi^+\pi^-2\pi^0$	[1.020–1.800]	$16.76 \pm 1.31 \pm 0.20$	$21.45 \pm 1.33 \pm 0.60$	-4.7 ± 1.8
$2\pi^+2\pi^-$	[0.800–1.800]	$14.21 \pm 0.87 \pm 0.23$	$12.35 \pm 0.96 \pm 0.40$	$+1.9 \pm 2.0$
$2\pi^+2\pi^-\pi^0$	[1.019–1.800]	$2.09 \pm 0.43 \pm 0.04$	–	–
$\pi^+\pi^-3\pi^0$ [3]	[1.019–1.800]	$1.29 \pm 0.22 \pm 0.02$	–	–
$3\pi^+3\pi^-$	[1.350–1.800]	$0.10 \pm 0.10 \pm 0.00$	–	–
$2\pi^+2\pi^-2\pi^0$	[1.350–1.800]	$1.41 \pm 0.30 \pm 0.03$	–	–
$\pi^+\pi^-4\pi^0$ [3]	[1.350–1.800]	$0.06 \pm 0.06 \pm 0.00$	–	–
$\eta(\to \pi^+\pi^-\gamma,\, 2\gamma)\pi^+\pi^-$	[1.075–1.800]	$0.54 \pm 0.07 \pm 0.01$	–	–
$\omega(\to \pi^0\gamma)\pi^0$	[0.975–1.800]	$0.63 \pm 0.10 \pm 0.01$	–	–
$\omega(\to \pi^0\gamma)(\pi\pi)^0$	[1.340–1.800]	$0.08 \pm 0.01 \pm 0.00$	–	–
K^+K^-	[1.055–1.800]	$4.63 \pm 0.40 \pm 0.06$	–	–
$K^0_S K^0_L$	[1.097–1.800]	$0.94 \pm 0.10 \pm 0.01$	–	–
$K^0K^\pm\pi^\mp$ [3]	[1.340–1.800]	$1.84 \pm 0.24 \pm 0.02$	–	–
$K\overline{K}\pi^0$ [3]	[1.440–1.800]	$0.60 \pm 0.20 \pm 0.01$	–	–
$K\overline{K}\pi\pi$ [3]	[1.441–1.800]	$2.22 \pm 1.02 \pm 0.03$	–	–
$R = \sum$ excl. modes	[1.800–2.000]	$8.20 \pm 0.66 \pm 0.10$	–	–
R [Data]	[2.000–3.700]	$26.70 \pm 1.70 \pm 0.03$	–	–
J/ψ	[3.088–3.106]	$5.94 \pm 0.35 \pm 0.00$	–	–
$\psi(2S)$	[3.658–3.714]	$1.50 \pm 0.14 \pm 0.00$	–	–
R [Data]	[3.700–5.000]	$7.22 \pm 0.28 \pm 0.00$	–	–
R_{udsc} [QCD]	[5.000–9.300]	$6.87 \pm 0.10 \pm 0.00$	–	–
R_{udscb} [QCD]	[9.300–12.00]	$1.21 \pm 0.05 \pm 0.00$	–	–
R_{udscbt} [QCD]	[12.0–∞]	$1.80 \pm 0.01 \pm 0.00$	–	–
$\sum\,(e^+e^- \to \text{hadrons})$	$[2m_{\pi^\pm}$–$\infty]$	$696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad}$	$711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{\rm SU(2)}$	$-14.7 \pm 7.9_{\rm tot}$

[1] e^+e^- data are used above 1.6 GeV (see [10]).
[2] Not including ω and ϕ resonances (see [10]).
[3] Using isospin relations (see [10]).

– The CMD-2 experiment is still the only one claiming systematic accuracies well below 1%. It is thus difficult to confront their data with results from other experiments. Whereas the measurements from OLYA are systematically lower than the new CMD-2 results in the peak region, there is a trend towards agreement above, as seen in Fig. 2. This behaviour appears to be confirmed by preliminary data from the KLOE experiment at Frascati using the radiative return method from the ϕ resonance [43]. We are looking forward to the final precise results from KLOE and from a similar analysis performed by the *BABAR* Collaboration under very different kinematic conditions [44].
The relative disagreement between older e^+e^- results and CMD-2 can be quantified using the CVC prediction: indeed the value of $(24.52 \pm 0.26_{\rm exp})\%$ obtained for $\mathcal{B}_{\rm CVC}(\tau^- \to \nu_\tau\pi^-\pi^0)$, reduces to $(23.69 \pm 0.68_{\rm exp})\%$ if the CMD-2 data are left out, increasing the relative difference with the measured value in τ decays from $(3.8 \pm 1.3)\%$ to $(7.4 \pm 2.9)\%$, a discrepancy hardly compatible with electromagnetic isospin breaking. Although the e^+e^- data are consistent with respect to

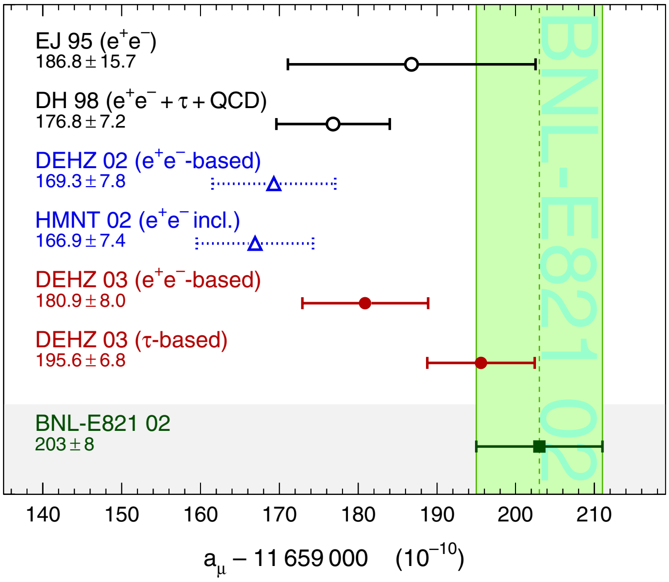

Fig. 4. Comparison of the results (9) with the BNL measurement [16]. Also given are our estimates [1,9] obtained before the CMD-2 data were available. For completeness, we show as triangles with dotted error bars the e^+e^--based results [10,15] derived with the previously published CMD-2 data [11]

the a_μ estimate within their systematic uncertainties, there is some evidence that the older data are pulling the value down.

– The most precise results on the τ $\pi\pi$ spectral function come from the ALEPH and CLEO experiments, operating in completely different physical environments. On the one hand, the main uncertainty in CLEO originates from the knowledge of the relatively low selection efficiency, a consequence of the large non-τ hadronic background, while the mass spectrum is measured with little distortion and good resolution. On the other hand, ALEPH has both large efficiency and small background, the main uncertainty coming from the π^0 reconstruction close to the charged pion, necessitating to unfold the measured spectrum from detector resolution and acceptance effects. A comparison of the τ spectral functions from ALEPH, CLEO and OPAL is given in Fig. 5. Agreement is observed within quoted errors, in particular in the high mass region, although CLEO results are a bit closer to e^+e^- data there. Overall, the τ data appear to be consistent.
– The last point concerns isospin corrections applied to the τ spectral functions. The basic components entering SU(2) breaking are well identified. The long-distance radiative corrections and the quantitative effect of loops have been addressed by the analysis of [14] showing that the effects are small. The overall effect of the isospin-breaking corrections (including FSR) applied to the τ $\pi\pi$ data, expressed in relative terms, is $(-1.8 \pm 0.5)\%$. Its largest contribution (-2.3%) stems from the uncontroversial short-distance electroweak correction [45]. One could question the validity of the chiral model used. The authors of [14] argue that the corrections are insensitive to the details of their model and essentially depend only on the shape of the pion form factor. As the latter is known from experiment to adequate accuracy, it seems difficult to find room for a $\sim 10\%$ effect as observed experimentally. Nevertheless, considering the situation regarding the first two experimental points, it would seem worthwhile to invest more theoretical work into the problem of isospin breaking.

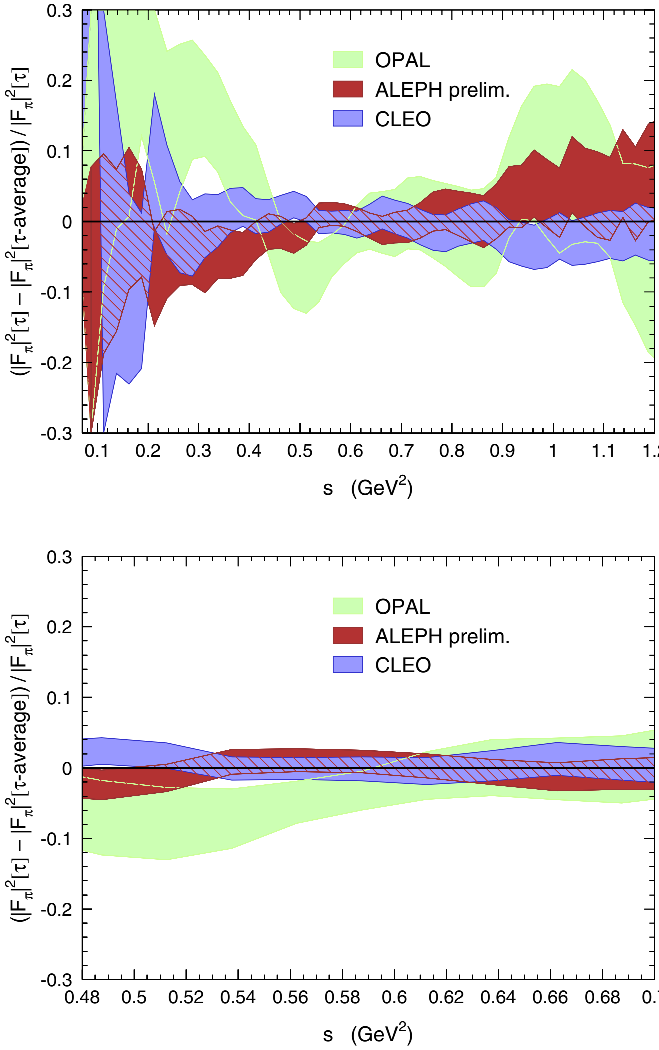

Fig. 5. Relative comparison of the $\pi^+\pi^-$ spectral functions extracted from τ data from different experiments, expressed as a ratio to the average τ spectral function. The lower figure emphasizes the ρ region

7 Conclusions

An update of our analysis of the lowest-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment has been performed following a reevaluation by the CMD-2 Collaboration of their e^+e^- annihilation cross sections. Part of the previous discrepancy between the e^+e^- and τ $\pi\pi$ spectral functions has now disappeared so that the corresponding evaluations of the lowest-order hadronic polarization contribution to the muon magnetic anomaly are closer. However, incompatible cross section measurements remain between 0.85 and 1 GeV so that we do not proceed with an average of the two evaluations. The e^+e^-- and τ-based predictions are respectively 1.9 and 0.7 standard deviations below the direct measurement from the g-2 Collaboration at BNL. The forthcoming results from radiative return with KLOE and *BABAR* will be decisive to sort out the remaining problems in the $\pi\pi$ and 4π spectral functions.

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